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Quantum Teleportation

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The purpose of this text is to introduce the concept of quantum teleportation. Quantum teleportation is a feature that arises from entanglement of two particles, each of which can be in one of two states — a qubit. The theory of quantum mechanics allows for the transmission of an arbitrary qubit state via a quantum communication channel, provided by an entangled qubit pair. Due to quantum teleportation's reliance on entangled qubits, it has a very close relation to the EPR paradox. John S. Bell's research on the EPR paradox has lead to the discovery that quantum mechanics violates local realism. It will be shown that, despite arising from a not completely understood concept of quantum mechanics, quantum teleportation does not violate the postulates of relativity or the quantum mechanical no-cloning and no-communication theorems. A crucial part of the importance of quantum teleportation is the fact that it has been experimentally achieved. A discussion about the experimental challenges will be given, many of which still need to be overcome to allow for an application of quantum teleportation in quantum computing.

Acknowledgements

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Contents

1 Introduction

The most fundamental difference between classical and quantum mechanics is the possibility of superposition. In quantum mechanics, a particle can be in an arbitrary number of states simultaneously, as long as they are allowed under given restrictions. The most famous "classical" analogy of this is Schrödinger's cat. Superposition brings with it a whole range of particle properties, which give rise to many potential applications, especially in quantum computing and encryption.
Quantum mechanics has the potential to answer some of the most fundamental questions in physics, although some aspects of the theory are not yet fully understood. In fact, Albert Einstein claimed that the theory of quantum mechanics was incomplete. He highlighted this in a paper, published in 1935, in collaboration with Boris Podolsky and Nathan Rosen [1]. They presented what is nowadays known as the EPR paradox, questioning the fundamentals of quantum mechanics. This paper has triggered a deep investigation of the foundations of quantum mechanics, and its impact on the nature of particles. Due to this research, not only has the understanding of quantum nature been driven to greater heights, but also new light has been shed on the restrictions of superposition.
One very significant application arising from superposition is quantum teleportation. Initially proposed as a gedanken experiment by Charles Bennet *et al.* [2] in 1993, quantum teleportation has been successfully performed experimentally [3, 4]. Quantum teleportation uses a pair of entangled particles, whose states are codependent, to transmit the state of a third particle instantaneously over an arbitrary distance. This suggests the possibility of a purely quantum mechanical communication channel. To complete the teleportation process, an additional, classical communication channel is required. This classical communication channel limits the transmission speed to the speed of light. Nonetheless, this particular application of the quantum nature of particles is of unquestionable significance.
An introduction to the basic concepts of measurement, qubits and entanglement is presented in Section 2. Emphasis is placed on explaining the nature of entangled particles and how measurements affect their combined state. Following this, in Section 3, an overview of the EPR paradox is given, including a short discussion of John S. Bell's work within this context. These sections introduce the foundation upon which the theory of quantum teleportation is built. Section 4 covers theoretical treatment of quantum teleportation, followed by a discussion about the difficulties which were overcome to realise the concept experimentally. Finally, a concluding word on the significance of quantum teleportation, possible applications and their limitations will be given in Section 5. This includes a discussion about the challenges that still need to be overcome to make any application of quantum teleportation possible.

2 Quantum States, Measurement and Entanglement

In quantum mechanics, the state of a particle is described by a vector in Hilbert space, $|\psi\rangle$, where Dirac bra-ket notation is used. We will only be concerned with states obeying the normalisation condition $\langle\psi|\psi\rangle = 1$, where $\langle\psi|\psi\rangle$ represents the inner product or overlap of the states. The *principle of superposition* states that, if the particle can be in either of the orthogonal states $|\psi\rangle$ or $|\phi\rangle$, then it may also be in the state $|\Phi\rangle = \alpha|\psi\rangle + \beta|\phi\rangle$, where $\alpha, \beta \in \mathbb{C}$, obeying the normalisation condition $|\alpha|^2 + |\beta|^2 = 1$ [5].

2.1 Measurements in Quantum Mechanics

In quantum mechanics, every physical quantity is represented by a Hermitian operator, $\hat{A}$. The operator applied to a wave function reveals information about the corresponding physical quantity. The eigenvectors of any Hermitian operator form a complete basis [5]. Therefore, we can represent any state in the basis of eigenvectors of an operator. Assuming a two dimensional basis for $\hat{A}$, $\{|a+\rangle, |a-\rangle\}$, any state can be written as

$$|\psi\rangle = \alpha\,|a+\rangle + \beta\,|a-\rangle\,, \tag{1}$$

where the eigenvalues of $|a\pm\rangle$ are $a_{\pm}$. A measurement of $\hat{A}$ on the state $|\psi\rangle$ can return either a_+ or a_-. The probability of obtaining a_+ is $|\langle a+|\psi\rangle|^2 = |\alpha|^2$, and similarly the probability of obtaining a_- is $|\beta|^2$. Therefore, one can imagine the particle being in the state $|a+\rangle$ with probability $|\alpha|^2$ or $|a-\rangle$ with probability $|\beta|^2$. Why this is not strictly true will be discussed in Section 3. If we obtain the result a_+, the new state of the particle instantaneously becomes $|a+\rangle$, so that if we measure $\hat{A}$ again, we will obtain a_+ with certainty. This is known as *collapse of the wave function* [5]. A consequence of this is that a measurement of a physical quantity is always performed in the basis of eigenstates of the corresponding operator. Therefore, the post-measurement state corresponds to one of the basis vectors. This is known as a *von Neumann measurement* [6].

If two operators, $\hat{A}$ and $\hat{B}$, both representing physical quantities, commute, i.e. $[\hat{A},\hat{B}] = 0$, they can share eigenstates and so, the physical quantities corresponding to these operators can be known precisely and simultaneously. This is impossible if the operators do not commute, i.e. $[\hat{A},\hat{B}] \neq 0$ [1,5]. Consider the spin operators in the x, y, z directions, $\hat{S}_x, \hat{S}_y, \hat{S}_z$, and the squared total spin operator $\hat{\mathbf{S}}^2 = \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2$. These operators obey the commutation properties $[\hat{S}_i, \hat{S}_j] \neq 0$ and $[\hat{\mathbf{S}}^2, \hat{S}_i] = 0$, where $i, j = x, y, z$ and $i \neq j$. Hence, one can never know two spin components of one particle simultaneously — only one component and the total magnitude of its spin. Therefore, $\hat{\mathbf{S}}^2$ and any $\hat{S}_i$ share simultaneous eigenstates [5]. If we measure an eigenstate of the $\hat{S}_z$ operator, $|\psi\rangle$, we obtain

$$\hat{\mathbf{S}}^2\,|\psi\rangle = s(s+1)\hbar^2\,|\psi\rangle \qquad \hat{S}_z\,|\psi\rangle = m_s\hbar\,|\psi\rangle\,, \tag{2}$$

where s and m_s are the spin quantum numbers. For a spin-$\frac{1}{2}$ state $s = \frac{1}{2}$ and $m_s = \pm\frac{1}{2}$. In a basis of the $\hat{S}_z$ operator the spin states in the z direction can be represented as

$$|{\uparrow}\rangle \to \begin{pmatrix}1\\0\end{pmatrix} \qquad |{\downarrow}\rangle \to \begin{pmatrix}0\\1\end{pmatrix}, \tag{3}$$

which correspond to $m_s = +\frac{1}{2}$ and $m_s = -\frac{1}{2}$, respectively. In this basis, all spin-$\frac{1}{2}$ component operators are represented as $\hat{S}_i \to \frac{\hbar}{2}\sigma_i$, where σ_i are the well known Pauli matrices,

$$\sigma_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} \qquad \sigma_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix} \qquad \sigma_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}. \tag{4}$$

All Pauli matrices are unitary, i.e. $\sigma_i\sigma_i = I$, where I is the identity matrix. These three matrices correspond to 180^o spin rotations about the x, y and z axes and therefore represent unitary transformations, which do not change quantum probabilities [7]. Eigenstates of the $\hat{S}_x$ and $\hat{S}_y$ operators can also easily be constructed:

$$\begin{aligned} |{\uparrow_x}\rangle &= \frac{1}{\sqrt{2}}(|{\uparrow}\rangle + |{\downarrow}\rangle) \qquad & |{\downarrow_x}\rangle &= \frac{1}{\sqrt{2}}(|{\uparrow}\rangle - |{\downarrow}\rangle) \\ |{\uparrow_y}\rangle &= \frac{1}{\sqrt{2}}(|{\uparrow}\rangle + i\,|{\downarrow}\rangle) \qquad & |{\downarrow_y}\rangle &= \frac{1}{\sqrt{2}}(|{\uparrow}\rangle - i\,|{\downarrow}\rangle). \end{aligned} \tag{5}$$

Each $\hat{S}_i$ then has two eigenstates corresponding to the eigenvalues $\pm\frac{\hbar}{2}$, as expected for a spin-$\frac{1}{2}$ particle. Spin states of such a particle can be determined using a Stern-Gerlach apparatus [5, 7].

From this point onwards, all spin-$\frac{1}{2}$ states are assumed to correspond to eigenstates of the $\hat{S}_z$ operator.

2.2 Qubits and Bell states

Two-level systems, such as the spin-$\frac{1}{2}$ state of a fermionic particle, where the particle can only be in the states $|\uparrow\rangle$ and $|\downarrow\rangle$, are referred to as *quantum bits*, or *qubits* [7]. The particle state can generally be expressed as

$$|\psi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle\,, \tag{6}$$

where $|\alpha|^2 + |\beta|^2 = 1$, as above. In contrast to classical bits, qubits exist in a superposition of $|\uparrow\rangle$ or $|\downarrow\rangle$, which gives rise to new attributes and possible applications [8]. For a two particle system, where the particles will be denoted by subscripts 1 and 2, respectively, the total state can be expressed as the tensor product of the individual states,

$$|\Phi\rangle = \sum_{j,k} c_{jk}\,|j_1\rangle \otimes |k_2\rangle = \sum_{j,k} c_{jk}\,|j_1 k_2\rangle\,, \tag{7}$$

where the summations over j and k run over all possible state combinations of particles 1 and 2 [7, 9]. Here $|j_1\rangle$ and $|k_2\rangle$ specify the state of particles 1 and 2, respectively, and $|j_1 k_2\rangle$ is a shorthand notation for the combined state. The constant c_{jk} is related to the probability of finding the system in a specific combined state. The individual particle states exist only in their own respective Hilbert space. States which can be written as a single tensor product of vectors in different Hilbert spaces, i.e. $|\Phi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$, are known as separable or pure states [10]. For such states, a measurement, determining the state of one particle has no effect on the state of the second particle. As an example, consider a two particle system in the state

$$|\Omega\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_2\rangle + |\uparrow_1\uparrow_2\rangle) = \frac{1}{\sqrt{2}}\,|\uparrow_1\rangle \otimes (|\downarrow_2\rangle + |\uparrow_2\rangle). \tag{8}$$

If we measure the state of particle 1, particle 2 can be in either state $|\downarrow_2\rangle$ or $|\uparrow_2\rangle$ and therefore the measurement of particle 1 has no effect on the state of particle 2.
On the contrary, we can now consider the combined spin states of two particles, 1 and 2,

$$\left|\Psi^+_{12}\right\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_2\rangle + |\downarrow_1\uparrow_2\rangle) \tag{9a}$$

$$\left|\Psi^-_{12}\right\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle) \tag{9b}$$

$$\left|\Phi^+_{12}\right\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\uparrow_2\rangle + |\downarrow_1\downarrow_2\rangle) \tag{9c}$$

$$\left|\Phi^-_{12}\right\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\uparrow_2\rangle - |\downarrow_1\downarrow_2\rangle). \tag{9d}$$

None of these states can be written as $|\psi_1\rangle \otimes |\psi_2\rangle$, and are therefore referred to as *entangled*. The four states displayed in Eqs. 9 are known as Bell states, forming a complete orthogonal basis. They are all maximally entangled, i.e. if we determine the state of one particle by measurement, the wave function collapses and we automatically know the state of the second particle [2, 7, 8].

3 EPR Paradox and Bell's Theorem

One of the most famous critics of quantum mechanics was Albert Einstein, who was convinced that the theory's formulation was incomplete. Together with Boris Podolsky and Nathan Rosen, Einstein formulated a gedanken experiment which is now known as the EPR paradox [1].

Einstein, Podolsky and Rosen (EPR) were concerned about the quantum mechanical description of the nature of entangled particles. If we consider an entangled particle pair, and measure one of the particle states, the wave function collapses and we instantaneously change the state of the second particle. A cornerstone of special relativity is that no information can travel faster than the speed of light, known as *relativistic causality*. Now, consider separating the entangled particles by a large distance, so no relativistically causal interaction between the particles is possible. Initially, before any measurement, both particles are in a superposition of states. If a measurement is made on one of the particles, determining its state, there is no way for the other particle to instantaneously know the new state of its entangled partner, or even that a measurement has occurred. This is known as *locality*. As the outcome of the measurement of the first particle is purely probabilistic in the theory of quantum mechanics, so is the state of the second particle. Hence, the theory allows for a reality in which both states of the second particle can exist simultaneously. This is a violation of *realism*, which requires the physical quantities of a system to be well defined. Einstein, Podolsky and Rosen expected any theory to obey *local realism*, which is the combination of both locality and realism. As the theory of quantum mechanics violates either one or the other, it was declared incomplete. Their suggested resolution to this problem was that information about exact particle states is encoded in local hidden variables (local to the particle), which needed to be included in the theory [1, 11].
In 1964, John S. Bell [12] published a paper, which distinguished between the possibility of local hidden variables in the formulation of quantum mechanics and the violation of local realism. Bell derived certain constraints, known as Bell's inequalities, which arise from allowing for local hidden variables in the theory of quantum mechanics. Therefore, a violation of these constraints would contradict the existence of hidden variables in quantum mechanics, suggesting a violation of local realism [13]. Bell's inequalities were found to be violated experimentally numerous times. One of the earlier experiments was performed by Weihs *et al.* [14]. A rigorous experimental proof is very difficult to achieve, due to possible loopholes. Nowadays, physicists are still working on new techniques to account for the last of these loopholes [15], but the violation of Bell's inequalities is now widely accepted. Therefore, a violation of local realism must occur in nature. Hence, the measured particle states are not predetermined on separation of the particles. This is not a violation of relativistic causality as no information can be transmitted via this method. The mathematical proof of this statement is known as the *no-communication theorem*, and is given in Ref. [16].

4 Quantum Teleportation

4.1 A theoretical viewpoint

In the last section we have explored the nature of entangled qubits and found that local realism was violated by quantum mechanics. The question arises as to the possibility of exploiting this violation in any way. It was already discussed that instantaneous information transfer via this *quantum communication channel* is impossible. However, in 1993, Charles Bennet *et al.* [2] proposed a groundbreaking gedanken experiment, in which entangled particles were used to "teleport" single qubit states across an arbitrary distance. This process became known as *quantum teleportation*. The significant feature of quantum teleportation is the reproduction of a single particle state, at an arbitrary distance from the original particle. A representative diagram of the process is shown in Fig. 1.
According to the *no-cloning theorem*, no exact copy of a quantum state can be produced, without destroying the original state. The proof of this theorem is given in Ref. [7]. Hence, it is expected that the original particle state gets destroyed in the quantum teleportation process.
Throughout this teleportation process, the well known duo Alice and Bob will be of great help. In this scenario they are separated by a long distance. Suppose Alice and Bob share an entangled qubit pair

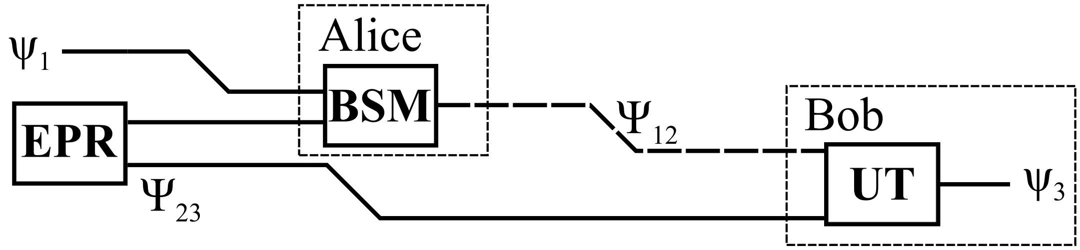

Figure 1: A schematic diagram of the quantum teleportation process is given here. First the entangled EPR particle pair is created (EPR) in the state $|\Psi_{23}\rangle$, of which one particle is given to Alice and one to Bob. Alice possesses another particle in the state $|\psi_1\rangle$, whose state she wants to teleport to Bob. Alice performs a Bell state measurement (BSM) on the combined state of her two particles, $|\Psi_{12}\rangle$. She then sends the result of her measurement to Bob via a classical communication channel (dashed line). Once Bob knows the outcome of Alice's measurement, he can perform a unitary transformation (UT) on the state of his particle to replicate the state $|\psi_1\rangle$, leaving his particle in state $|\psi_3\rangle$ [2, 7, 8].

in one of the Bell states from Eq. 9, say [1]

$$\left|\Psi_{23}^{+}\right\rangle = \frac{1}{\sqrt{2}}(|\uparrow_2\downarrow_3\rangle + |\downarrow_2\uparrow_3\rangle). \tag{10}$$

This entangled pair consists of two particles, where particle 2 belongs to Alice and particle 3 belongs to Bob. Alice has another qubit, particle 1, in the state

$$|\psi_1\rangle = \alpha\,|\uparrow_1\rangle + \beta\,|\downarrow_1\rangle\,, \tag{11}$$

which she keeps completely isolated from particle 2. Alice wants to teleport the state $|\psi\rangle$ to Bob, where $|\psi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle$ represents only a state, without specification of which particle it belongs to.
The overall state of the three particles is the product state

$$\begin{aligned}|\Omega_{123}\rangle &= |\psi_1\rangle \otimes \left|\Psi_{23}^{+}\right\rangle \\ &= \frac{\alpha}{\sqrt{2}}(|\uparrow_1\uparrow_2\downarrow_3\rangle + |\uparrow_1\downarrow_2\uparrow_3\rangle) + \frac{\beta}{\sqrt{2}}(|\downarrow_1\uparrow_2\downarrow_3\rangle + |\downarrow_1\downarrow_2\uparrow_3\rangle),\end{aligned} \tag{12}$$

where, for example, $|\uparrow_1\uparrow_2\downarrow_3\rangle \equiv |\uparrow_1\rangle \otimes |\uparrow_2\rangle \otimes |\downarrow_3\rangle$. This overall product state can be projected into the Bell basis $\{\left|\Psi_{12}^{+}\right\rangle, \left|\Psi_{12}^{-}\right\rangle, \left|\Phi_{12}^{+}\right\rangle, \left|\Phi_{12}^{-}\right\rangle\}$,

$$\begin{aligned}|\Omega_{123}\rangle = \frac{1}{2}\Big[&\left|\Psi_{12}^{+}\right\rangle \otimes (\alpha\,|\uparrow_3\rangle + \beta\,|\downarrow_3\rangle) + \left|\Psi_{12}^{-}\right\rangle \otimes (\alpha\,|\uparrow_3\rangle - \beta\,|\downarrow_3\rangle) \\ &+ \left|\Phi_{12}^{+}\right\rangle \otimes (\alpha\,|\downarrow_3\rangle + \beta\,|\uparrow_3\rangle) + \left|\Phi_{12}^{-}\right\rangle \otimes (\alpha\,|\downarrow_3\rangle - \beta\,|\uparrow_3\rangle)\Big].\end{aligned} \tag{13}$$

Hence, the overall system could be found in any of the four states

$$\left|\Psi_{12}^{+}\right\rangle \otimes (\alpha\,|\uparrow_3\rangle + \beta\,|\downarrow_3\rangle) \tag{14a}$$
$$\left|\Psi_{12}^{-}\right\rangle \otimes (\alpha\,|\uparrow_3\rangle - \beta\,|\downarrow_3\rangle) \tag{14b}$$
$$\left|\Phi_{12}^{+}\right\rangle \otimes (\alpha\,|\downarrow_3\rangle + \beta\,|\uparrow_3\rangle) \tag{14c}$$
$$\left|\Phi_{12}^{-}\right\rangle \otimes (\alpha\,|\downarrow_3\rangle - \beta\,|\uparrow_3\rangle), \tag{14d}$$

with equal probability $\frac{1}{4}$. All the states of particle 3 are unitary transformations of the state $|\psi\rangle$, corresponding to the Pauli matrices. We can therefore express Eq. 13 as

$$|\Omega_{123}\rangle = \frac{1}{2}\Big[\left|\Psi_{12}^{+}\right\rangle \otimes |\psi_3\rangle + \left|\Psi_{12}^{-}\right\rangle \otimes (\hat{\sigma}_z\,|\psi_3\rangle) + \left|\Phi_{12}^{+}\right\rangle \otimes (\hat{\sigma}_x\,|\psi_3\rangle) - i\left|\Phi_{12}^{-}\right\rangle \otimes (\hat{\sigma}_y\,|\psi_3\rangle)\Big], \tag{15}$$

[1] The process works just as well with any other initial Bell state.

where the factor $-i$, in the last term, is just an irrelevant phase factor. Up to this point, the system has not been interfered with at all, simply the mathematical representation of the states has been manipulated.
Now, Alice performs a measurement on particles 1 and 2, to determine which Bell state they are in. During this process, the initial state of particle 1, $|\psi_1\rangle$, will be destroyed, as required by the no-cloning theorem. The wave function in Eq. 15 will instantly collapse and project particle 3 into the state corresponding to Alice's measurement result. At this point Bob does not have any information about the state of particle 3 — he does not even know that a measurement has occurred. Particle 3 can be in any of the states $|\psi_3\rangle$, $\hat{\sigma}_z|\psi_3\rangle$, $\hat{\sigma}_x|\psi_3\rangle$ or $\hat{\sigma}_y|\psi_3\rangle$. It is only when Alice tells Bob the outcome of her measurement, that Bob will have full information about which of the four states his particle is in. By then transforming the state of particle 3, according to the corresponding Pauli operator, Bob can produce an exact replica of the state $|\psi\rangle$. This completes the teleportation process [2, 7, 8].
During this process relativistic causality was not violated. No information was instantaneously transmitted from Alice to Bob. Even though Alice determined the Bell state of particle 1 and 2, and therefore had information about the state of particle 3, Bob did not. Only after Alice tells Bob her results will he know the state of particle 3. The exchange of information about the measurement results needs to occur via a classical communication channel, which is limited to the speed of light. Therefore, quantum teleportation is not a proof of the violation of local realism and does not contribute to the resolution of the EPR paradox. Theoretically, the two particles could have communicated during the time period required to transmit Alice's measurement results to Bob.

4.2 Experimental realisation

Quantum teleportation has successfully been performed experimentally, using linearly polarized photons. The experimental realisation of quantum entanglement is much more practical with photons, as one can use optical methods to produce entangled photon pairs and perform Bell measurements. Also, transmitting photons over large distances, without interaction, is much easier than transmitting particles. In 2012, X. Ma *et al.* [3] successfully teleported quantum states over a distance of 143km between two Canary Islands. In 2017, J. Ren *et al.* [4] managed to perform quantum teleportation between an earth based observation station in Ngari, Tibet and the low orbit Micius satellite over a distance of 1400km.
All arguments introduced with spin-$\frac{1}{2}$ particles, including the operator formulation from Section 2.1, are valid for any qubit system. Linear photon polarization can have similar properties to the discussed spin states. The photons used are polarized either vertically or horizontally, with respect to an arbitrary orientation. For clarity, we change the previous spin state label "$\uparrow$" to "$\updownarrow$" for vertical polarization, and "$\downarrow$" to "$\leftrightarrow$" for horizontal polarization. For photons, the Bell states, from Eqs. 9, read

$$|\Psi^{\pm}_{12}\rangle = \frac{1}{\sqrt{2}}(|\updownarrow_1\leftrightarrow_2\rangle \pm |\leftrightarrow_1\updownarrow_2\rangle), \tag{16a}$$

$$|\Phi^{\pm}_{12}\rangle = \frac{1}{\sqrt{2}}(|\updownarrow_1\updownarrow_2\rangle \pm |\leftrightarrow_1\leftrightarrow_2\rangle). \tag{16b}$$

The most important stages of the experimental realisation of quantum teleportation are discussed below. A broad overview is given on how to create an entangled photon pair (EPR), the difficulties of Bell state measurements (BSM), and the procedure of the final unitary transformation of the state of Bob's particle (UT), as in Fig. 1.

4.2.1 Entangled photon pair creation

A linearly polarized, entangled photon pair can be created using the *type-II spontaneous parametric down-conversion method*, where type-II corresponds to opposite polarization of resulting photons [17].

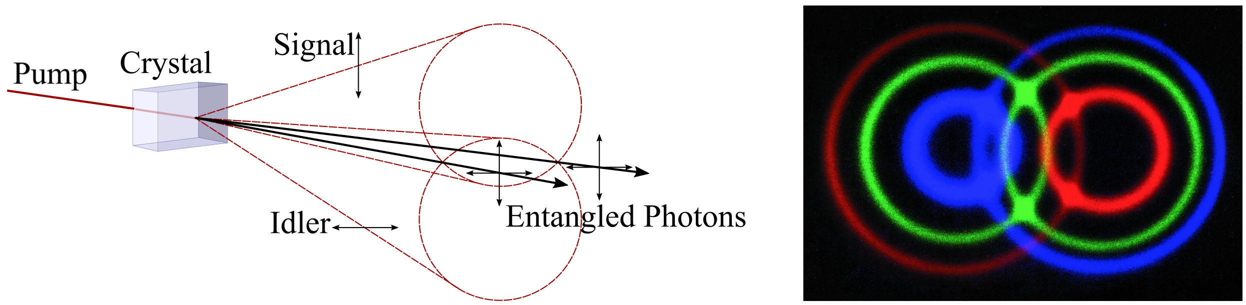

Figure 2: *Left:* A laser beam is used to pump a nonlinear, birefringent crystal, producing two cones of horizontally and vertically polarized photons (signal and idler). The double-headed arrows indicate polarization direction. Entangled pairs are found at the intersections of the cones [18]. *Right:* A photograph showing the intersecting cones of orthogonally polarized photons emitted from a beta barium-borate crystal. A continuous spectrum of frequencies is emitted from the crystal, i.e. an infinite set of cones, each corresponding to a different frequency and opening angle. This is expected from the phase matching conditions in Eq. 18 and Eq. 19. Filters were applied to make only some cones visible, as in this photograph. The pairing of small and large cone opening angles in the case of photons represented by blue and red rings, is a consequence of phase matching [19]. *Photo Credit: [20]*

A schematic of the process is shown in Fig. 2. In a nonlinear, birefringent, optical medium, such as a barium-borate crystal, the refractive index is dependent on both polarization and direction of photon propagation, relative to the crystal's optic axis. The optic axis of a birefringent crystal is the direction in which the refractive index is independent of polarization [21]. Such a crystal is pumped with laser light. The crystal polarization density $\mathbf{P}$, due to the electric field of the pumping photons, $\mathbf{E}$, i.e. the induced dipole moment per unit volume, is given by [22]

$$P_i = \epsilon_0 \left(\sum_j \chi^{(1)}_{ij} E_j + \sum_{k,l} \chi^{(2)}_{ikl} E_k E_l + \dots \right). \tag{17}$$

Here, P_i stands for any of the crystal polarization density components, $E_{j,k,l}$ represents any of the electric field components of the pumping photons and $\chi^{(n)}$ is the n^{th} order susceptibility tensor. The summations run over the spatial components x, y, z. The higher order susceptibility tensors give rise to the birefringent properties of the crystal. The interaction of the pumping field, $\mathbf{E}$, with the second order susceptibility term can lead to the spontaneous creation of orthogonally polarized photons, obeying energy and momentum conservation. The quantum mechanical details of this process are given in Ref. [19]. The process can be thought of as an excitation of the crystal medium, where one pump photon is absorbed and its energy is then re-emitted in terms of two photons, instead of one. The photons are usually referred to signal (s) and idler (i), due to historic reasons. This so called *down-conversion* occurs for approximately one out of $10^7 - 10^{11}$ pumping photons [19], so it is a very rare process. Energy conservation imposes the restriction

$$\omega_p = \omega_s + \omega_i \tag{18}$$

on the photons, where ω_p, ω_s and ω_i are the angular frequencies of the pumping, signal and idler photons, respectively. Momentum conservation implies that the relation

$$\mathbf{k}_p = \mathbf{k}_s + \mathbf{k}_i \tag{19}$$

holds, where $\mathbf{k}_p$, $\mathbf{k}_s$ and $\mathbf{k}_i$ are the wave vectors of the pumping, signal and idler photons, respectively [19]. It is important to emphasise that the pumping, signal and idler photons each feel different refractive indices in the crystal due to their polarizations and directions of propagation, relative to the crystal optic axis. They therefore travel with different velocities in the crystal. Both momentum and energy conservation for the non-collinear signal and idler photons is only possible in the presence of a birefringent material. Otherwise, the above restrictions in Eq. 18 and Eq. 19, often referred to as phase matching conditions [19], would be incompatible. Signal and idler photons are emitted into two separate cones with a relative spatial shift, as shown in Fig. 2. The opening angle of the cones is dependent on the frequency of the emitted photons and the relative shift depends on the angle of the pump beam with the crystal optic axis [19, 23]. The relation between frequency and angular width of the cones is clearly visible in a photograph, from an experiment by Paul Kwiat and Michael Reck at the University of Innsbruck, Austria, in 1995 [17], in Fig. 2. Entangled photon pairs are emitted at the intersection of the cones, as one photon must be from the signal cone and the other from the idler cone, but only measurement can distinguish them [18]. The created photons will therefore be in the state

$$|\Psi_{12}^{\alpha}\rangle = \frac{1}{\sqrt{2}}(|\updownarrow_1 \leftrightarrow_2\rangle + e^{i\alpha} |\leftrightarrow_1 \updownarrow_2\rangle), \tag{20}$$

where the relative phase α depends on the birefringent properties of the crystal [17]. This method can be used to generate either of the entangled states $|\Psi_{12}^{\pm}\rangle$, by a corresponding choice of crystal properties. The other two Bell states, $|\Phi_{12}^{\pm}\rangle$, can be created by inserting a half wave plate into one of the photon paths. The half wave plate will change the polarization of any photon from horizontal to vertical and vice versa [17].

The creation of photon pairs through this method is not without difficulties. One arising issue is that the photons travel with different speeds in the nonlinear crystal and will therefore exit the crystal with a short time delay. This is a problem, as it makes the photons distinguishable. This issue can be resolved by changing the polarization of both photons from horizontal to vertical and vice versa after they have exited the crystal, and then sending both through a second crystal of the same type. This is known as walk-off compensation [17]. The photon which has travelled through the first crystal faster will travel slower in the second one, and vice versa for the other photon. The time delay issue is therefore resolved and complete indistinguishability is restored. Another difficulty of this method is finding the optimal pump rate to minimise the probability of producing two photon pairs at the same time. This is controlled by using femto-second lasers with a very short pulse duration as the pumping source [3, 24].

Purely deterministic photon creation methods are still in development [18]. Nonetheless, type-II spontaneous parametric down-conversion is relatively easy to implement experimentally and is therefore widely used in quantum experiments [25].

4.2.2 Bell state measurement

Experimentally, there is a way to distinguish between Bell states using optical equipment. However, this method comes with a major drawback.

Only the Bell state $|\Psi^-\rangle$ is antisymmetric under exchange of photons; all other states are symmetric. Photons are bosons, so their overall wave function (polarization times spatial) must be symmetric. Photons in the singlet polarization state $|\Psi^-\rangle$ will be paired with an antisymmetric spatial wave function, to make the overall wave function symmetric. On the contrary, the triplet states $|\Psi^+\rangle$ and $|\Phi^{\pm}\rangle$ are already symmetric and will be paired with a symmetric spatial wave function. We can treat the action of a beamsplitter on the spatial wave function only, since in each of the Bell states horizontal or vertical polarization is equally likely for each photon. Consider the beamsplitter in Fig. 3, where inputs are labelled a and b and outputs are labelled c and d. If we have no information about which photon enters which input of the beamsplitter, symmetric and antisymmetric spatial wave functions

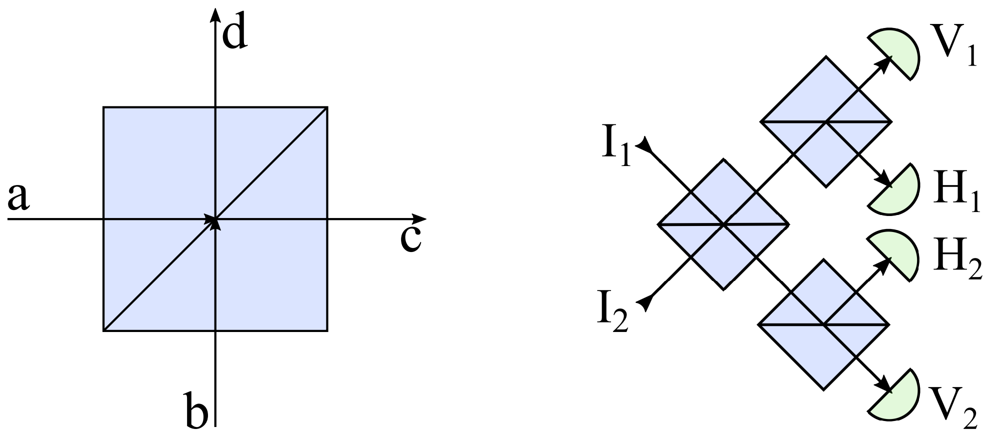

Figure 3: *Left:* A single beamsplitter with inputs a and b and outputs c and d. Photons enter the beamsplitter via a or b, get transmitted or reflected according to their polarization, and exit via c or d. *Right:* The two photons enter a system of three beamsplitters via input I_1 and I_2, respectively. The beamsplitters are set up to transmit vertically polarized photons and reflect horizontally polarized photons. Dependent on their Bell state, the photon pairs will take different paths through the beamsplitters, and will then trigger either of the photon detectors at H_1, H_2 or V_1, V_2, where H and V measure horizontally and vertically polarized photons, respectively. Depending on the combination of triggered detectors, the Bell states $|\Psi^{\pm}\rangle$ can be distinguished.

are represented as

$$\left|B^{+}\right\rangle = \frac{1}{\sqrt{2}}(|a_1 b_2\rangle + |b_1 a_2\rangle) \tag{21a}$$

$$\left|F^{-}\right\rangle = \frac{1}{\sqrt{2}}(|a_1 b_2\rangle - |b_1 a_2\rangle), \tag{21b}$$

where the labels B and F stand for boson and fermion, respectively. The state $|a_1 b_2\rangle$ represents photon 1 and photon 2 entering the beamsplitter via input a and input b, respectively, and similarly for the other states. The analogy to bosons and fermions is valid, as these spatial wave functions on their own can be thought of as obeying bosonic or fermonic statistics, respectively. Fermions obey the Pauli exclusion principle, which states that two fermions will never occupy the same state. Hence, a pair of photons in the initial state $|F^{-}\rangle$ will never exit the same beamsplitter outputs. Bosons do not obey the exclusion principle, so the opposite argument can be applied to the state $|B^{+}\rangle$. Therefore, photons in the initial state $|B^{+}\rangle$ will always exit the beamsplitter from the same output [26].

This argument is also confirmed by a mathematical treatment. The considered beamsplitter transforms spatial states according to

$$|a\rangle \rightarrow \frac{1}{\sqrt{2}}(|c\rangle + i\,|d\rangle) \tag{22a}$$

$$|b\rangle \rightarrow \frac{1}{\sqrt{2}}(|d\rangle + i\,|c\rangle), \tag{22b}$$

where reflected photons undergo a phase shift of $\frac{\pi}{2}$, which is incorporated in the factor i. Under these transformations, the states $|B^{+}\rangle$ and $|F^{-}\rangle$ become

$$\left|B^{+}\right\rangle \rightarrow \frac{i}{\sqrt{2}}(|c_1 c_2\rangle + |d_1 d_2\rangle) \tag{23a}$$

$$\left|F^{-}\right\rangle \rightarrow \frac{1}{\sqrt{2}}(|c_1 d_2\rangle - |d_1 c_2\rangle), \tag{23b}$$

after the photons have exited the beamsplitter, which concludes the proof of the above argument. Photons in the initial state $|B^+\rangle$ both exit the beamsplitter either via port c or port d with 50\50 probability. Photons in the initial state $|F^-\rangle$ always exit via opposite ports [26].
After sending the photon pair through the first beam splitter, we can distinguish the state $|\Psi^-\rangle$ from the states $|\Psi^+\rangle$, $|\Phi^\pm\rangle$. Photons in the state $|\Psi^-\rangle$, with the antisymmetric spatial wave function $|F^-\rangle$, will exit the first beamsplitter via opposite ports. For the states $|\Psi^+\rangle$ and $|\Phi^\pm\rangle$, both photons will exit the same port. Having distinguished between the symmetric and antisymmetric polarization states, we can easily distinguish $|\Psi^+\rangle$ from $|\Phi^\pm\rangle$. This is done by placing another beam splitter at each output port of the first beam splitter, as shown in Fig. 3. Each photon will exit one of the four possible exit ports of the second set of beamsplitters, triggering one of the four photon detectors connected to each output. If the initial state is $|\Psi^-\rangle$, the two photons will either trigger H_1, V_2 or V_1, H_2, as the photons are orthogonally polarized. If the initial state is $|\Psi^+\rangle$, detectors H_1, V_1 or H_2, V_2 will be triggered. On the contrary, for either of the states $|\Phi^\pm\rangle$, both photons will trigger only one of the four detectors, as they exit the first beamsplitter via the same port and have the same polarization. Therefore, there is no way of distinguishing the states $|\Phi^\pm\rangle$ with this method [3, 4, 27].
The above method allows us to distinguish $|\Psi^+\rangle$ from $|\Psi^-\rangle$ and both of these from $|\Phi^\pm\rangle$. Not being able to distinguish $|\Phi^\pm\rangle$, limits the efficiency of Bell state measurements to 50%. In fact, this is the largest achievable efficiency using linear optical equipment to teleport photon polarization [28, 29]. A mathematical proof of this statement is given in Ref. [29]. The above method is most commonly used in experiments, as its implementation is very simple [3, 4]. In principle, complete Bell state measurements are possible, which would make quantum teleportation significantly more efficient. These methods use nonlinear optics [30]. The highest experimentally achieved efficiency of a complete Bell state measurement is still low, compared with methods using linear optics [30, 31].

4.2.3 Unitary transformation

After excluding the states $\left|\Phi_{12}^{\pm}\right\rangle$, the remaining possible states of Alice's and Bob's three qubit system, as given in Eq. 14, are now

$$\left|\Psi_{12}^{+}\right\rangle \otimes (\alpha\,|\updownarrow_3\rangle + \beta\,|\leftrightarrow_3\rangle) \tag{24a}$$

$$\left|\Psi_{12}^{-}\right\rangle \otimes (\alpha\,|\updownarrow_3\rangle - \beta\,|\leftrightarrow_3\rangle). \tag{24b}$$

When Alice measures the Bell state $\left|\Psi_{12}^{+}\right\rangle$, Bob's photon is already in the desired state and hence the teleportation process is complete. However, if Alice measures the state $\left|\Psi_{12}^{-}\right\rangle$, Bob must apply a π phase shift between the horizontal and vertical polarization components. This corresponds to the unitary transformation given by $\hat{\sigma}_z$. Experimentally, this can be achieved with a half wave plate, which introduces a π phase shift between wave components travelling parallel and perpendicular to the plate's fast axis [21]. After this unitary transformation, the teleportation process is complete.

5 Conclusion

The successful realisation of experimental quantum teleportation leads to big discussions about its possible range of applications. As we have seen, quantum teleportation is a very delicate process, with many specific requirements. This includes the generation of entangled qubits, which then need to be separated by a large distance without interfering with the particle states, and the low efficiency Bell state measurement. Nonetheless, the most likely field of application is quantum computing. Quantum teleportation could be applied to message encryption, for example. The process requires two different communication channels: one quantum — the reaction of one entangled qubit to the Bell state measurement of the other — and one classical — the transfer of information about the result of the

Bell state measurement [2]. This way of transmitting encrypted messages is extremely secure, as the entangled particles can only exist between sender and receiver. Even if the the classical communication channel is interfered with and information about the measurement result leaks to a third party, it is not much use without the receiver's entangled particle. Additionally, the entangled particle state can never be copied, without destroying the state of the original one, due to the no-cloning theorem. The transmission process would most likely use photons as entangled particles, as these travel fast and are relatively easy to transmit, without state interference. If they were interfered with, the teleportation would fail, as the entangled state would be destroyed, which is another security advantage.

A quick and efficient way of producing entangled photon pairs would be necessary for fluent data transmission via teleportation, as mentioned in Section 4.2.1. Otherwise, one faces the consequence of very slow and energy inefficient transmission. An additional challenge is the storage of teleported qubit states. It is crucial for any quantum computer to store qubits, otherwise quantum computers would have no advantage over classical computers. The property that photons travel at the speed of light makes them perfect for transmission, but not for storage. Therefore, the photon state must be transformed into a storable qubit. Such qubit storages are referred to as quantum memories [32]. The integration of quantum teleportation with quantum memories is still to be achieved [29]. Furthermore, the current efficiency limit of 50% for polarization teleportation, as mentioned in Section 4.2.2, is probably the most severe drawback of quantum teleportation at the moment and therefore the first problem that needs to be overcome to make it implementable in any real life applications.

Taking the presented arguments further, one could, theoretically, "teleport" objects. This would require teleporting the quantum state of every constituent of the object individually. Teleporting the quantum state of each particle of a macroscopic system is an incredibly difficult task, mainly as the quantum states change due to interactions between the object's constituents. Only isolated quantum states remain unchanged and can be teleported. Even if this was achievable, this form of teleportation would still be limited by the speed of light [33].

This text also highlights the consistency of quantum mechanics with other fundamental physical laws. John S. Bell's discussion on the EPR paradox [12] suggested a violation of local realism in quantum mechanics. This suggestion was of great danger to Einstein's theory of relativity, which was constructed from the idea that nothing can travel faster than the speed of light, known as Einstein's second postulate. Hence, if this postulate was violated, one would be forced to question Einstein relativity. One of the beauties of quantum mechanics lies in the consistency with Einstein's second postulate. Even though quantum mechanics violates local realism, the fact that no information can be sent solely via the quantum mechanical communication channel of entangled particles makes the two theories compatible. Quantum teleportation is a wonderful example of a violation of local realism, but also a consistency with Einstein's second postulate.

Overall, many challenges need to be overcome to make quantum teleportation at all applicable, which is unlikely to happen within the next few years. This is acceptable as quantum computers will not replace classical computers in the foreseeable future. I strongly believe that quantum teleportation will play an essential role in the eventual implementation of efficient quantum computers, and by then the challenges mentioned above will be overcome. Quantum teleportation should also be recognised for its experimental representation of the special features, that arise from superposition in the theory of quantum mechanics. At this point in time, a complete interpretation of quantum mechanics has not been found, but experimental realisations like these bring humanity one step closer to understanding the very fundamentals.

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